Q1. (a) $y = \frac{e^{-x^3}}{x}$

let $y = \frac{f(x)}{g(x)}$

$f'(x) = e^{-x^3}(-3x^2)$

$g'(x) = 1$

$y' = \frac{f'(x)\,g(x) - f(x)g'(x)}{(g(x))^2}$

$= \frac{(e^{-x^3})(-3x^2)(x) - (e^{-x^3})(1)}{x^2}$

$= \frac{(e^{-x^3})(-3x^3 - 1)}{x^2}$

$y' = -\frac{(e^{-x^3})(3x^3+1)}{x^2}$

(b) $y = e^{t^2} + 3e^{-t}$

$y' = e^{t^2}(2t) + (3e^{-t})(-1)$

$= 2te^{t^2} - 3e^{-t}$ //

Q2. if $y = \frac{5}{2}\left(e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right)$, then $y'' = \frac{y}{25}$

$$y' = \frac{5}{2}\left(\left(\frac{1}{5}\right)e^{\frac{x}{5}} + \left(-\frac{1}{5}\right)e^{-\frac{x}{5}}\right)$$

$$= \frac{1}{2}e^{\frac{x}{5}} - \frac{1}{2}e^{-\frac{x}{5}}$$

$$= \frac{1}{2}\left(e^{\frac{x}{5}} - e^{-\frac{x}{5}}\right)$$

$$y = \frac{5}{2}\left(e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right)$$

$$\boxed{\frac{2}{5}y = e^{\frac{x}{5}} + e^{-\frac{x}{5}}}$$

$$y'' = \frac{1}{2}\left(\left(\frac{1}{5}\right)e^{\frac{x}{5}} - \left(-\frac{1}{5}\right)e^{-\frac{x}{5}}\right)$$

$$= \frac{1}{10}e^{\frac{x}{5}} + \frac{1}{10}e^{-\frac{x}{5}}$$

$$y'' = \frac{1}{10}\left(e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right)$$

$$y'' = \frac{1}{10}\left(\frac{2}{5}y\right)$$

$$y'' = \frac{2}{50}y$$

$$\boxed{y'' = \frac{y}{25}} \ //$$

Q3. $N(t) = 1000\,[30 + e^{-\frac{t}{30}}]$

(a) the initial number = ?

At time, $t = 0$

$$N(0) = 1000\,[30 + e^{-\frac{0}{30}}]$$

$$= 1000\,[30 + 1]$$

~~initial number = ...~~

initial number = 31000 bacteria

(b) rate of change = $N'(t)$ = ?

$$N'(t) = 1000\left[0 + \left(-\frac{1}{30}\right)\left(e^{-\frac{t}{30}}\right)\right]$$

$$N'(t) = 1000\left(-\frac{e^{-\frac{t}{30}}}{30}\right)$$

(c) rate when $t = 20$?

$$N'(20) = 1000\left(-\frac{e^{-\frac{20}{30}}}{30}\right) = -17.113 \text{ bacteria/hour}$$

(c) The ~~b~~ bacteria population is decreasing at the rate of ≈17.1 bacteria per

(d) largest number on $0 \leq t \leq 50$

$N'(t) < 0$

$N'(t) = 0$

$1000\left(-\frac{e^{\frac{t}{30}}}{30}\right) = 0$

$\left(-\frac{e^{\frac{t}{30}}}{30}\right) = 0$

~~$e^{\frac{t}{30}} = 0$~~

$N'(t) = 0$ is undefined so there has no zeros.

∴ At end point

$N(0) = 31000$ bacteria (from (a))

$N(50) = 1000\left[30 + e^{-\frac{50}{30}}\right]$

$= 1000\left[30 +\quad\right]$

$= 30188.87$ bacteria

∴ the largest bacteria number is 31000 at the very begining at $t = 0$.

(e) to find out when time passes, ~~let~~ we set limit as $t \to \infty$.

$\lim_{t \to \infty} 1000\left[30 + e^{-\frac{t}{30}}\right]$

$= \lim_{t \to \infty} 1000\left[30 + e^{-\frac{\infty}{30}}\right]$

$= 1000[30 + 0]$

$= 30000$

As time passes, the ~~most~~ number of bacteria decrease and approches a horizontal asymptote at 30000.